Prototyping and load testing of thin-shell concrete floors

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Abstract
Buildings are being constructed at ever faster rates, fuelled by population growth and urbanisation. The total worldwide floor area of buildings is expected to almost double over the next 40 years, the equivalent of constructing Paris every five days [1]. The majority of the mass and embodied energy (60% to 70%) in a typical multi-storey building structure exists within the floors [2], making these a primary target for sustainable structural design. In a typical reinforced concrete slab, much of the concrete is assumed to be cracked and therefore not structurally utilised, but nevertheless adds significant weight. This project proposes a radical re-design of concrete floors, using precast textile-reinforced concrete shells with an in-situ foamed concrete fill. By harnessing membrane action, self-weight savings of 62% have been demonstrated for typical spans (compared to traditional flat slabs).

This paper discusses the design, optimisation, construction, measurement, analysis and structural testing of two prototype shells, one with and one without foamed concrete fill. The construction accuracy was quantified using digital scanner measurements which were then used as a geometry input for the analysis model. Each shell was loaded both uniformly and asymmetrically to beyond the design loading before failure. The foamed concrete was found to provide only a small increase in strength and stiffness. A design methodology for full-scale, practical application is currently under development.

Keywords: optimisation, concrete shells, floors, construction, structural testing, sustainable design.

1. Introduction
Reducing the consumption of building materials is vital if the construction industry is to meet sustainability targets, such as a legally binding 80% reduction in greenhouse gas emissions required in the UK by 2050. A comprehensive study by the Green Construction Board estimates that carbon emissions associated with construction materials must be reduced by 39% from 2010 levels to meet this target, in tandem with even greater reductions in operational emissions [3]. Despite this, levels of embodied greenhouse gas emissions remain strongly linked to overall construction output, with no overall downward trend in recent decades [4]. The adoption of radical new technologies and construction practices is therefore increasingly urgent.

The structure contains the majority of the material in a typical multi-storey building, of which the floors typically contribute 60% to 70% [2]. For centuries, arches and vaults, acting in compression, were the only way to create spanning structures from solid, durable, yet brittle materials such as stone, brick and concrete. The invention of steel reinforced concrete allowed the design of horizontal beams and slabs, acting in bending, which today dominate multi-storey construction. Concrete bending elements also have the greatest potential for reduction in material use, since typically most of the concrete is assumed to be cracked and hence contributes very little structurally (whilst nonetheless adding weight). The fluidity of concrete creates limitless geometric possibilities, which are habitually
ignored by casting in prismatic moulds. However, modern materials, manufacturing methods and computational techniques are increasingly allowing designers to explore and rediscover the structural advantages of non-planar geometries. This project proposes a vaulted floor structure with the primary aim of minimising embodied energy through structural efficiency, whilst also achieving architectural performance and constructability for widespread application.

2. Structural concept

The proposed structural system is shown in Figure 1. The primary structure is a pre-cast textile reinforced concrete (TRC) shell. TRC is a composite material consisting of a fine-grained concrete with layers of fibre reinforcement in the form of an open orthogonal mesh. The flexibility of the reinforcement allows the production of complex or freeform geometries, and the glass or carbon fibres typically used do not require concrete cover for durability, thus minimising the required shell thickness. Lightweight foamed concrete is cast on the precast shell in-situ to create a level floor surface, as well as providing good thermal and acoustic insulation [5]. Using a shell without stiffeners simplifies the manufacturing process by allowing the use of single-sided formwork and creates better opportunities for service integration within the structural depth by removing potential obstructions.

In some cases, the lateral thrust required to support the vaults can be provided by walls, neighbouring vaults, or the ground. However, a grid of steel ties spanning between the columns is proposed for general multi-storey applications. Since lateral movement of the supports causes bending of the shell, the stiffness of the tie affects the shells’ performance. The ties can also be prestressed to reduce the stresses in the shell under maximum loading.

Figure 1: Concept for a thin-shell vaulted floor structure.

In a previous study a variety of candidate shell geometries, including hyperbolic paraboloids, groin-vaults and geometries form-found using dynamic relaxation, were compared for their structural performance [6]. A parabolic groin-vault was identified as an efficient structural form despite its singly-curved geometry, which further simplifies formwork construction. The required volume of fill was also found to be low. Using a groin-vaulted shell of 50mm thickness, 8m span and 800mm total depth, reductions in embodied energy and self-weight were demonstrated of 62% and 64% respectively compared to an equivalent steel reinforced flat slab for a typical office design scenario. The weight savings also reduce column and foundation loads, potentially halving the structural material requirements for a tall concrete building.

3. Materials and design process

Two physical prototype shells were constructed and tested at the University of Bath, with the aim of investigating the structural behaviour, verifying the analysis and design methodology, and examining materials and construction processes. The two structures were identical in design apart from the presence of the foamed concrete fill. With a span of 2m and rise of 0.2m, the prototypes were designed
To represent a typical design for an office or residential building, as described in the previous study [6], at a quarter scale. The design thickness of 18mm was determined using the method described in this section. By scaling all dimensions proportionally and retaining the full-scale loadings, the stresses and materials remain representative of the full-scale structure. The shells were designed to support maximum and minimum loadings of 10.31kN/m² and 3.75kN/m² respectively, based on a live load of 3.50kN/m² and superimposed dead load of 1.00kN/m², with partial factors of 1.50 and 1.35 respectively. A full-scale self-weight of 2.75kN/m² was assumed, with additional loading applied to make up for the self-weight lost in the scaling process. The corner supports were 62.5mm square, each representing a quarter of the area of a typical 500mm square column at full scale.

For design, concrete properties matching a C32/40 mix were assumed, with a strength and stiffness of 32MPa and 33GPa respectively [7]. The reinforcement was a woven textile made with alkali-resistant glass fibres coated with an epoxy resin. Tensile tests were performed to determine the strength and stiffness properties of the yarns, which differ in each direction due to variations in construction and yarn spacing. The properties of the weakest direction were chosen for the design of the shell, with a cross-sectional area of 52.2mm²/m, a stiffness of 55.7GPa and an ultimate tensile strength of 1326MPa. A single top and bottom layer of reinforcement was assumed with 3mm of cover. The material properties of the concrete and reinforcement were used to calculate utilisation of based on combinations of axial and bending force as described in Hawkins, et al. [6].

A design procedure was developed to determine the required shell thickness, tie diameter, tie pre-strain and finally the geometry of the shell itself. The groin-vault geometry was defined by a two-dimensional curve starting at the edge of the column and ending at the mid-span, the highest point of the vault. The parabola previously chosen to define the groin vault has no physical justification since there is no pure compression solution for a cross-vault supported at its corners, even considering a simple uniform loading scenario. The geometry was therefore parameterised using a Bézier curve with four control points defined by the half-span (L), rise (H) and two parameters a and b (Figure 2).

The shell was split into quartile regions and load patterns of all distinct combinations of these were considered, as shown in Figure 2. It is important to consider the maximum and minimum total loading since, with the application of tie prestress, these result in the largest inward and outward horizontal displacement of the supports. Each of the loading patterns were found to maximise utilisation across various local regions of the shell.

![Figure 2: Parameterisation and live loading patterns used for geometry optimisation (all dimensions in mm).](image)

The bending and axial forces were found using a linear finite element (FE) analysis with the foamed concrete fill ignored. After manually selecting a shell thickness and tie rod diameter, the three remaining variables (a, b and tie pre-strain) were optimised using a genetic algorithm to minimise the total bending strain energy envelope (the sum of the local maxima from each load pattern). Bending strain energy was found to be a reliable proxy for strength utilisation but requires fewer assumptions to calculate. Once the geometry and tie pre-strain had been optimised, the envelope of shell utilisation was calculated and inspected. The shell thickness or tie diameter were then modified until a satisfactory solution was found.
The chosen design features an 18mm thick shell with tie rods of 14.2mm$^2$ cross-sectional area (equivalent to a 16mm outer diameter threaded rod). The optimal geometric parameters were found to be $a=0.385$ and $b=0.326$, with a tie pre-strain of 0.56mm/m. With this optimised geometry, the total bending strain energy envelope was reduced by 9.8% compared to the starting parabolic profile.

A more detailed non-linear FE analysis was subsequently performed to determine the critical live load pattern. Live loading applied over one half of the shell only was found cause failure at the smallest load, with failure predicted at 7.5kN/m$^2$ due to the onset of cracking and subsequent loss of stability. The physical test setup was therefore designed to allow independent loading on each half of the shell.

### 4. Construction

The timber formwork consisted of rigid triangular frames with stiffeners supporting a thin (3mm) curved plywood forming surface. Four identical units were constructed that were joined along the diagonal edges (Figure 3), thus allowing easier construction, transportation and, crucially, striking and removal of the formwork after casting.

The corner supports were constructed from welded steel plates, and the formwork supported on screw jacks to allow precise positioning and levelling. A fine-grained concrete mix was developed with a maximum aggregate size of 2mm. This was applied by hand in three layers of 3, 12 and 3mm thickness, with reinforcement between each. Each of the two reinforcement layers consisted of four triangular segments with a 50mm overlap between the diagonal edges. Achieving a uniform thickness was a significant challenge. To minimise thickness variations, the concrete was weighed out evenly across each eighth of the shell before spreading and spot depth measurements were taken regularly.

For the filled shell, foamed concrete was then cast onto the TRC with the formwork still in place. Unlike the concrete used in the shell construction, the fluidity of the foamed concrete mix enabled straightforward construction of a level top surface. Compression tests on four foamed concrete cubes (cast during construction of the second shell) determined an average strength of 0.91MPa, Young’s modulus...
modulus of 0.203GPa and density of 806kg/m³. A sand/cement ratio of 1.0 was used, with a water/cement ratio of 0.5 and approximately 600ltr/m³ of foam for the target density.

After removing the formwork, the steel tie rods were inserted through holes in the bases of the corner supports. Each tie was fixed at one end but free to slide at the other (with a holding bolt), thus allowing pre-strain to be applied by rotating the bolt and measured by the angle of rotation.

Although accuracy had been prioritised throughout the construction process, some error in both the thickness of the shell and its centreline was expected. Both the formwork and top surface of each shell were therefore measured using a hand-held digital scanner, thus allowing a detailed assessment of the as-built geometry including both the thickness and the deviation of the mid-surface from the design (Figure 4). For each shell, the top and bottom scans were registered using the corner supports as a reference, since these remained fixed throughout construction.

The measured average thicknesses were 20.4mm (standard deviation 2.9mm) and 18.6mm (standard deviation 2.7mm) for the unfilled and filled shells respectively. Increased thickness at the corners and along the diagonals is visible, which results in a corresponding shift of the mid-surface away from that designed. Deviations in the mid-surface are also visible at the edges of the shell, where the thin plywood forming surface is most liable to deflect due to being supported only by the ribs.

![Figure 4: Measured thickness (left) and centreline error (right) for the unfilled shell.](image)

5. **Testing**

The shells were tested in-situ fourteen days after casting. Each corner was supported on PTFE sheets to allow free horizontal sliding, and vertical loading was applied via four hydraulic jacks which were controlled in pairs. The force from each jack was distributed to four loading points via fully pinned beam assemblies creating sixteen loading points, as shown in Figure 5. These were built up to 200mm square using plaster and evenly spaced across the shell to approximate a distributed area load. Instrumentation included a vertical transducer at the mid-span, eight vertical transducers at loading points over one half of the shell, eight horizontal transducers in orthogonal pairs at each support, a pair of strain gauges on each tie rod and several concrete strain gauges on the shell itself.

Four fine-grained concrete cubes were cast and tested with each shell, giving average strengths of 35.5MPa and 39.0MPa for the unfilled and filled shells respectively.
The test was carried out in five stages as summarised in Figure 6. Firstly, a uniform load was applied up to the dead load. The tie pre-strain was then applied by tightening the holding bolts, under a constant vertical load. Additional live loading was then applied uniformly and removed. Finally, the live load was increased over one half of the shell only until failure.

6. Results & analysis

The measured mid-span displacement from shell each test is shown in Figure 7. The foamed concrete fill appears to have had a stiffening effect, although the apparently higher strength of the concrete in the filled shell may also have contributed towards a higher stiffness. The maximum load reached in Phase 3 for the unfilled shell was reduced to 6.2kN/m$^2$ due to concerns about premature failure arising due to delamination of the bottom cover layer of the TRC near the supports. It was later determined that this was a result of voids in the concrete around the reinforcement (an issue rectified in the construction of the filled shell).

The non-linear FE model was updated using stress-strain relationships for the TRC, foamed concrete and steel ties as constructed, derived from separate physical tests. The scanned shell geometry was also imported into the FE model by individually modifying node locations and element thicknesses, allowing the effect of construction tolerances to be investigated. The resulting change of stiffness (measured in Phase 1) compared to the as-designed geometry was +1.8% and -6.5% for the unfilled and filled shells respectively. This means that, relative to the respective average shell thickness errors of +13.4% and +3.5%, the manufacturing errors appear cause some loss of stiffness.

The tie forces and horizontal support displacements were predicted with good accuracy by the FE model. The predicted mid-span displacement, included in Figure 7 for the filled shell, shows a greater stiffness than that measured in Phase 1. This discrepancy is currently under investigation.
Table 1 compares the measured and predicted ultimate strengths of each shell for the final asymmetric test phase. The filled shell has a higher strength than the unfilled shell in both the test and predictions. As well as benefiting from the fill, the concrete of the filled shell was stronger than the unfilled shell although the average thickness was 1.8mm less. The relative contribution of each of these factors will be the subject of more detailed future investigations. The predicted strengths were consistently lower than those measured in testing. It was noticed in the simulations that instability occurred very shortly after the onset of significant cracking (evident from the linearity of the FE model results in Figure 7), contrasting with the cracking observed in the tests. Altering the post-cracking behaviour of the TRC in the models may therefore yield more accurate results. Modifying the geometry of the FE model to match the scan measurements resulted in reductions in predicted strength of 8.7% and 3.4% for the filled and unfilled shells respectively, indicating some geometric sensitivity and demonstrating the impact of construction tolerances.

Table 1: Comparison of measured and predicted ultimate strength.

<table>
<thead>
<tr>
<th></th>
<th>Estimated self-weight [kN/m²]</th>
<th>Ultimate asymmetric load [kN/m²]</th>
<th>Test</th>
<th>Predicted FEA (designed geometry)</th>
<th>Predicted FEA (measured geometry)</th>
</tr>
</thead>
<tbody>
<tr>
<td>unfilled shell</td>
<td>0.43</td>
<td>14.5</td>
<td></td>
<td>12.6</td>
<td>11.5</td>
</tr>
<tr>
<td>filled shell</td>
<td>0.79</td>
<td>15.8</td>
<td></td>
<td>14.6</td>
<td>14.1</td>
</tr>
</tbody>
</table>

The mode of deformation at failure can be seen for the filled shell in Figure 5, and was similar for both specimens. Distinct regions of hogging and sagging curvature were observed extending across the full width of the shell, closely matching the deformation predicted by the analysis.

7. Future work
The results from the physical testing have informed and verified the FE modelling approach, which can now be used with confidence to explore the possible applications and real-world performance of the shells. The practical design complexities of service openings, irregular floorplans and lateral stability requirements will also be investigated. This will culminate in real-world case studies and detailed performance comparisons with traditional flooring systems, including the potential effects of reduced weight on downstream building elements such as columns and foundations.
Constructing the shells by hand, whilst possible, was found to be slow and labour intensive. Preliminary research into a fully automatic construction process for thin TRC shells is currently underway by the authors, using robotic placement of both concrete and reinforcement. Automated manufacturing allows ‘mass customisation’ of precast elements, enabling greater architectural flexibility and structural efficiency. Significant reductions in construction tolerance, time and cost are also expected. Furthermore, the thickness of the shell and placement of the reinforcement could be optimised without additional manufacturing complexity, leading to further potential material savings.

8. Conclusions
This paper has introduced a novel, lightweight concrete flooring system and described the design, construction and testing of two prototypes. The chosen groin-vault geometry was parameterised and optimised based on multiple loading scenarios, leading to a 9.8% reduction in bending strain energy compared to a parabolic vault. The formwork, TRC shells and foamed concrete fill were constructed by hand and the deviations from the design geometry were measured. The shells were tested to destruction and found to have a moderately higher ultimate strength than predicted by non-linear FE modelling. The mechanism of failure closely matched the simulations in both tests. The presence of the foamed concrete fill was found to have only a small impact on the strength and stiffness, suggesting that it’s exclusion is a valid simplification for analysis.

The results verify the analysis model and provide further evidence of the high structural efficiency of the proposed system. Considerable savings in embodied materials, of potentially half over a full building structure, can now be demonstrated with confidence as the project progresses towards detailed case-studies and practical implementation.

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